When little things matter: Thoughts from the study of climate dynamics

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Sensitivity to initial conditions

With the Newtonian revolution in mechanics, scientists were given the ability, in theory, of reducing any dynamical system to a set of differential equations. Some speculated that if we could somehow take a near-perfect "snapshot" in time of all state variables of a given system, we could then integrate the differential equations describing that system and predict any future state.

This dream of mechanistic determinism was laid to rest (at least for complex systems) by the work of Edward Lorenz (1963). Lorenz studied a very simple set of equations describing convection in a fluid:

$$dX/d\tau = -\sigma X + \sigma Y$$
$$dY/d\tau = -XZ + rX - Y$$
$$dZ/d\tau = XY - bZ$$

where X, Y, and Z are the state variables of the system, τ is non-dimensional time, and σ, r, and b are adjustable parameters. The system has a stable and trivial solution at (X, Y, Z) = (0, 0, 0).

Lorenz found that the solution of these deterministic differential equations:

- Exhibited a type of regularity, but
- Was highly sensitive to the exact value of the initial conditions.

Figs. 1a and 1b shows the timeseries of Y for 40 units of time for two slightly different initial conditions. Although for the first 18 units of time there appears to be no difference between the two cases, as time progresses the two solutions diverge. The divergence is not entirely wild; the solutions have similar patterns. Yet the solution is completely different. The similarity in patterns, the regularity Lorenz found, is most clearly seen in a plot in time of all solutions of (X, Y, Z); the regions where the solution concentrates are called "strange attractors" (Fig. 1c).

The second finding, that small differences in initial condition can result in large differences in solution later on, is termed "sensitivity to initial conditions." This sensitivity is one reason why it is probably impossible to make accurate weather predictions beyond a few weeks.

(a)

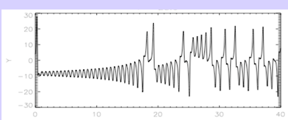
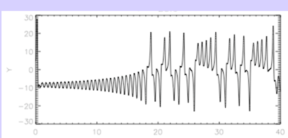


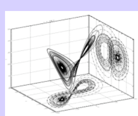
Fig. 1. Timeseries of Y for initial condition of
(a) (X, Y, Z) = (0, 4, 0);
(b) (X, Y, Z) = (0, 5, 0).

(c) shows the full phase-space for initial condition (X, Y, Z) = (0, 5, 0).

(b)



(c)



Stochastic scale-interactions

The motion of the atmosphere consists of structures at a variety of different sizes, from tiny turbulent eddies to global scale gyres like the Arctic and Antarctic circumpolar jets. But in a global climate model, the grid sizes are on the order of 100 x 100 km, and thus can only explicitly resolve large-scale atmospheric motions. Smaller phenomena must be approximated as a function of the large-scale state variables. This approximation is called parameterization.

Convection from cumulus clouds mixing and heating the troposphere is one phenomena that is almost always parameterized, since such clouds are much smaller than a model grid box and yet play a vital role in transporting energy in the atmosphere. Fig. 2 shows a schematic of an ensemble of clouds in a grid box. A cumulus parameterization assumes that the effect of all these clouds in the grid box can be specified by a single value (an "ensemble mean") of precipitation for the entire grid box.

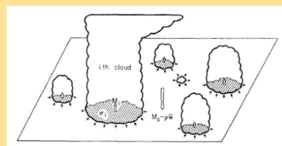


Fig. 2. Schematic of a single climate model grid box showing an ensemble of different types of cumulus clouds (from Arakawa & Schubert 1974).

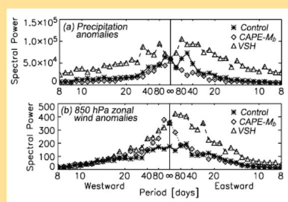


Fig. 3. Impact of stochastic convective schemes (CAPE-M_b and VSH curves) on tropical wavenumber 1 variability as simulated in the National Center for Atmospheric Research Community Climate Model 3 (Lin & Neelin 2003).

This assumption, however, is questionable, as it does not capture the large variability of the sub-grid cloud motions. Clouds are constantly changing. This variability might have an effect on the large-scale, but how to represent it in a climate model?

One answer is to represent this unresolved variability as a stochastic or random process. In work by myself with David Neelin (2003), we added Gaussian noise to the cumulus parameterization of a global climate model. Fig. 3 shows the effect of such noise upon large-scale (wavenumber 1) atmospheric phenomena in the tropics. Even though the noise has a timescale of 1 day, the noise enhances spectral power at timescales of months. This suggests even very small-scale variability, currently unaccounted for in climate models, can feed back on variability at global scales.

Implications

Throughout nature we see examples of small things having large impacts. Two such phenomena are resonance, which is used by a child on a swing, and "tipping points," as are found in epidemics. Climate science provides two additional phenomena: sensitivity to initial conditions and stochastic scale-interactions. What are some implications we can draw from these examples? We list some in the box below:

- **Limits of linear superposition:** Sometimes we cannot break up a system, analyze each part, and reassemble those parts to obtain the behavior of the complete system.
- **The limited value of the mean:** Sometimes the mean of a state variable does not capture the full-effect of the state variable on other phenomena.
- **Nonlinearity of cause and effect:** The amplitude of cause is not necessarily correlated with amplitude of effect (i.e. the race does not always go to the swift or the fight to the strong).
- **Perturbation to a new state:** One way small things can affect large is by moving the entire system to a new state, where the local "neighborhood" of possible states is entirely different.
- **Emergent properties:** Weakly- or non-correlated "random" phenomena can combine to form structures and variability with coherence at large-scales.
- **Nonlocality of scale-interactions:** Phenomena do not only interact with other phenomena at similar scales, but they also interact (directly or indirectly) with phenomena at different scales. Scale-interactions can go both up and downscale.
- **Boundaries of knowledge:** Sensitivity to initial conditions implies a likely inherent limit on our understanding of the dynamics of the climate system.
- **Humility:** Component interactions and sensitivity of solutions are not straightforward.

From these examples from the field of climate dynamics, it appears that God has created the world in such a way that little things do matter, in ways that can be exciting and unexpected. Perhaps in this way creation mirrors the Biblical principle that "... many who are first shall be last, and the last first" (Mk 10:31, NIV).

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References

Arakawa, A. and W. H. Schubert, 1974: Interaction of a cumulus cloud ensemble with the large-scale environment, Part I, *J. Atmos. Sci.,* **31,** 674–701.
Lin, J. W.-B. and J. D. Neelin, 2003: Toward stochastic deep convective parameterization in general circulation models, *Geophys. Res. Lett.,* **30,** 1162, doi:10.1029/2002GL016203.
Lorenz, E. N., 1963: Deterministic nonperiodic flow, *J. Atmos. Sci.,* **20,** 130–141.